Unit 5

Karnaugh Maps

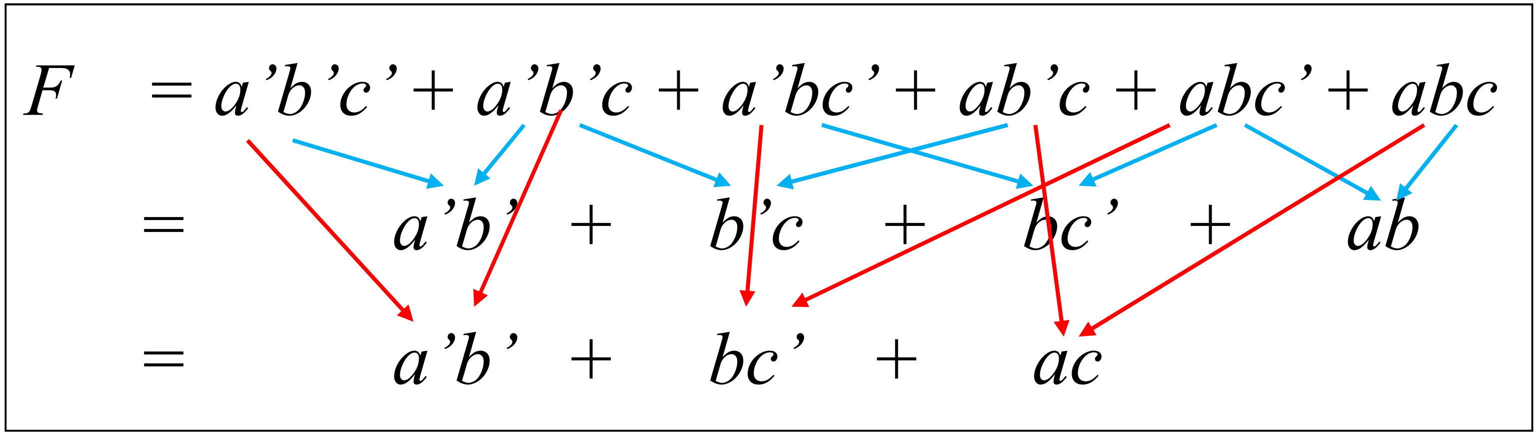
Outline

- Minimum forms of switching (Boolean) functions
- 2 and 3 variables Karnaugh maps
- 4 variables Karnaugh maps
- Minimum forms with essential prime implicants
- 5 variables Karnaugh maps

Minimum Forms of Switching Functions (1/2)

K-Map: A method to simplify Boolean function: **Faster, simpler & optimum solution.**

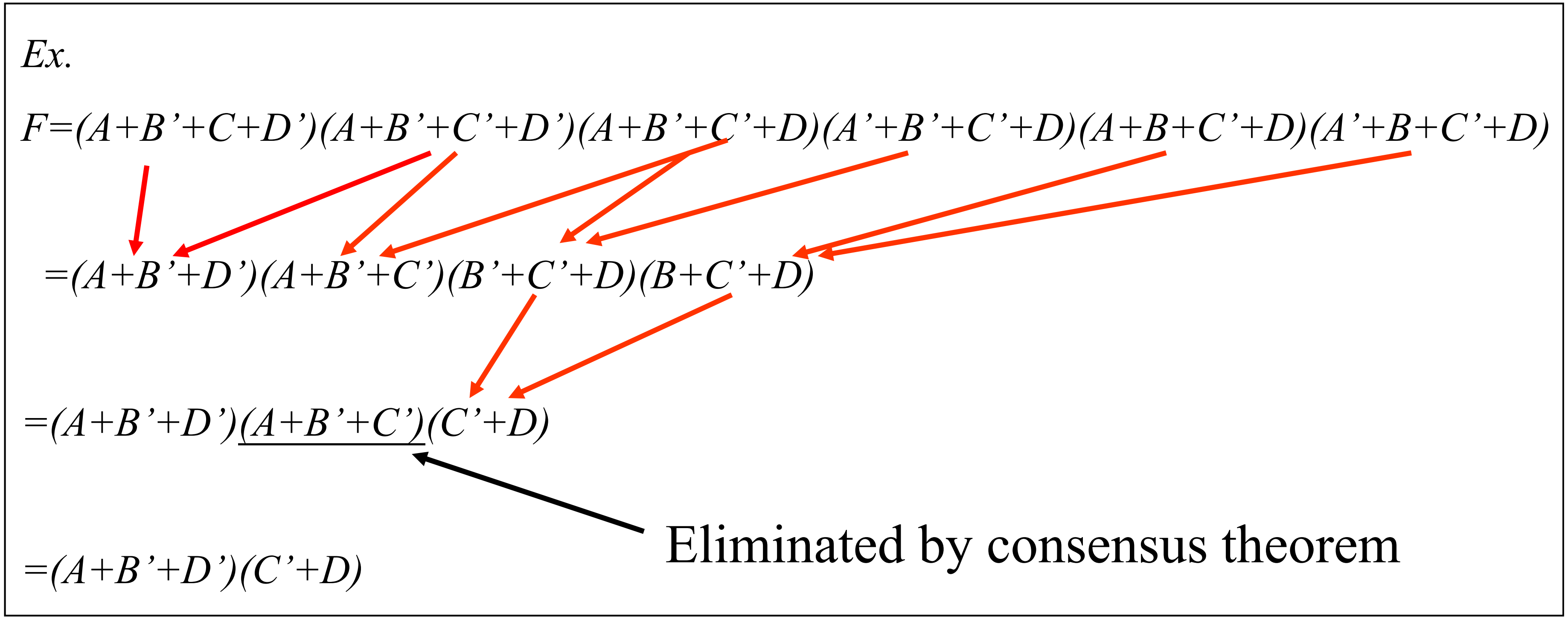
Minimum Number of Terms. → Minimum Sum of Products ← Minimum Number of Literals.



$$
\begin{aligned}
F &= a'b'c' + a'b'c + a'bc' + ab'c + abc' + abc \\
&= a'b' + b'c + bc' + ab \\
&= a'b' + bc' + ac
\end{aligned}
$$

Minimum Forms of Switching Functions (2/2)

Minimum Number of Factors. → Minimum Product of Sums ← Minimum Number of Literals.



Ex.

$F=(A+B'+C+D')(A+B'+C'+D')(A+B'+C'+D)(A'+B'+C'+D)(A+B+C'+D)(A'+B+C'+D)$

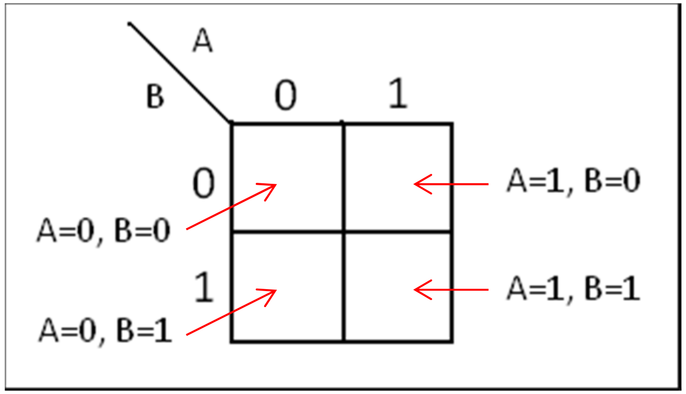
$=(A+B'+D')(A+B'+C')(B'+C'+D)(B+C'+D)$

$=(A+B'+D')\underline{(A+B'+C')}(C'+D)$

Eliminated by consensus theorem

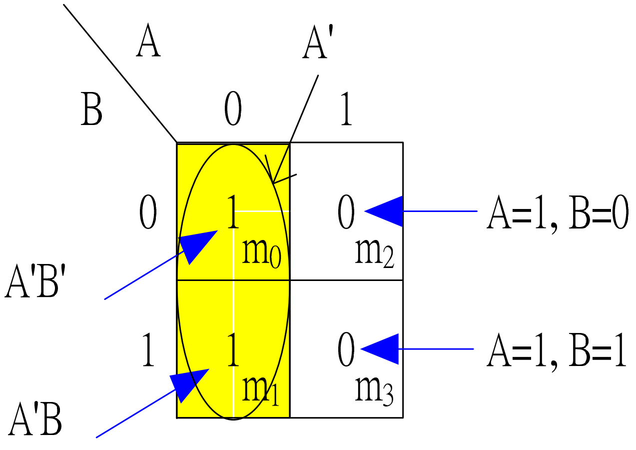
$=(A+B'+D')(C'+D)$

2 and 3 Variables Karnaugh Maps (1/8)



A	B	F
0	0	1
0	1	1
1	0	0
1	1	0

⇒

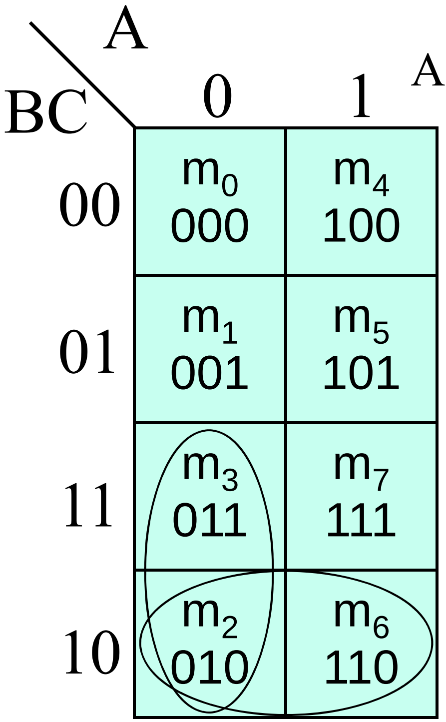


$$F = A'B'+A'B = A'(B'+B) = A'$$

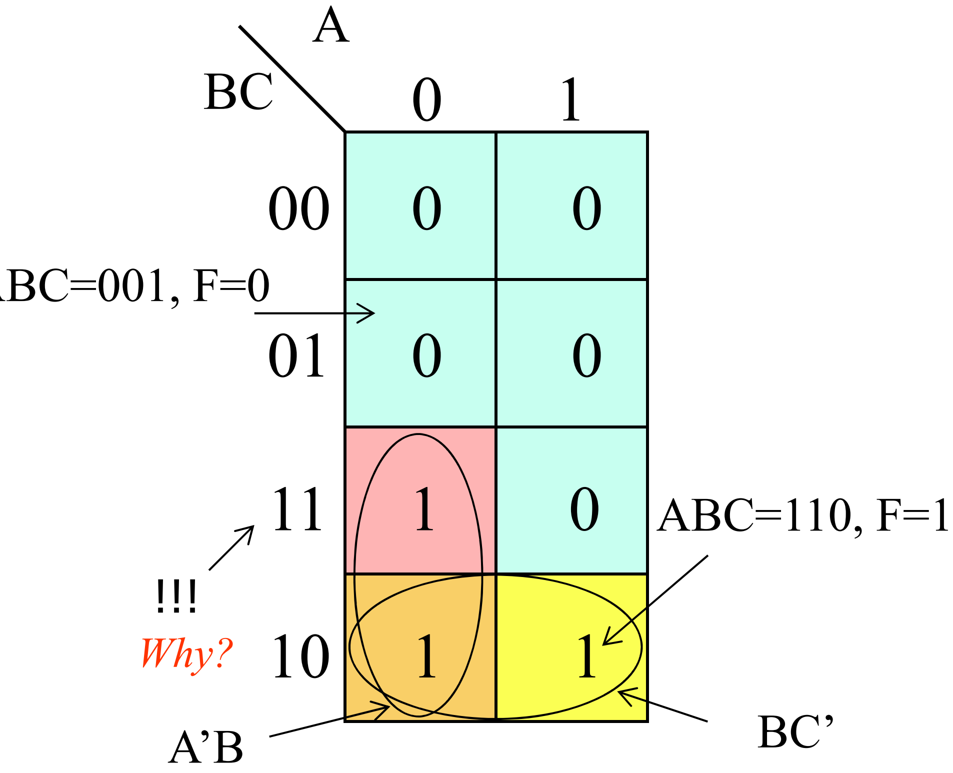
2 and 3 Variables Karnaugh Maps (2/8)

Ex 1:

A	B	C	F
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	0



BC \ A	0	1
00	m_0 000	m_4 100
01	m_1 001	m_5 101
11	m_3 011	m_7 111
10	m_2 010	m_6 110

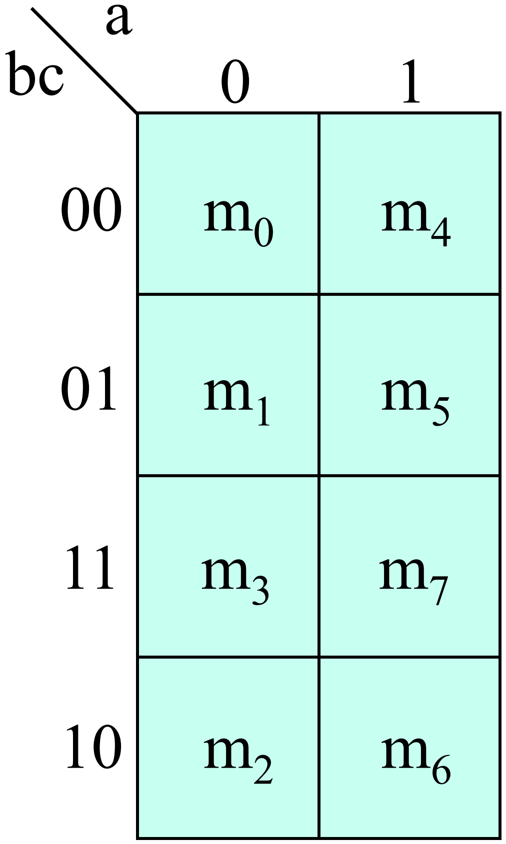


BC \ A	0	1
00	0	0
01	0	0
11	1	0
10	1	1

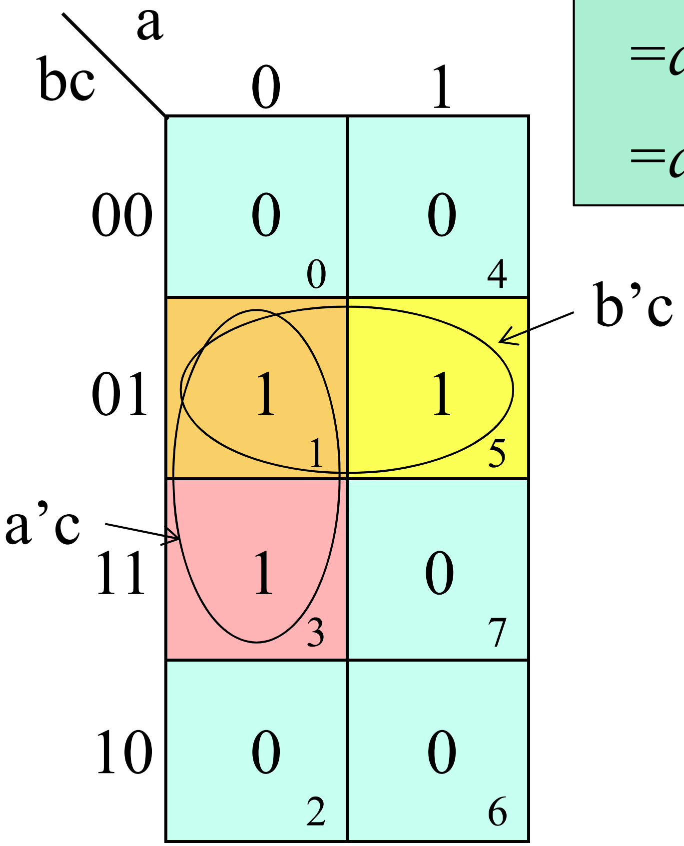
$$F=\Sigma m(2,3,6)$$
$$=A'BC'+A'BC+ABC'$$
$$=A'B+BC'$$

2 and 3 Variables Karnaugh Maps (3/8)

Ex 2:



bc \ a	0	1
00	m_0	m_4
01	m_1	m_5
11	m_3	m_7
10	m_2	m_6

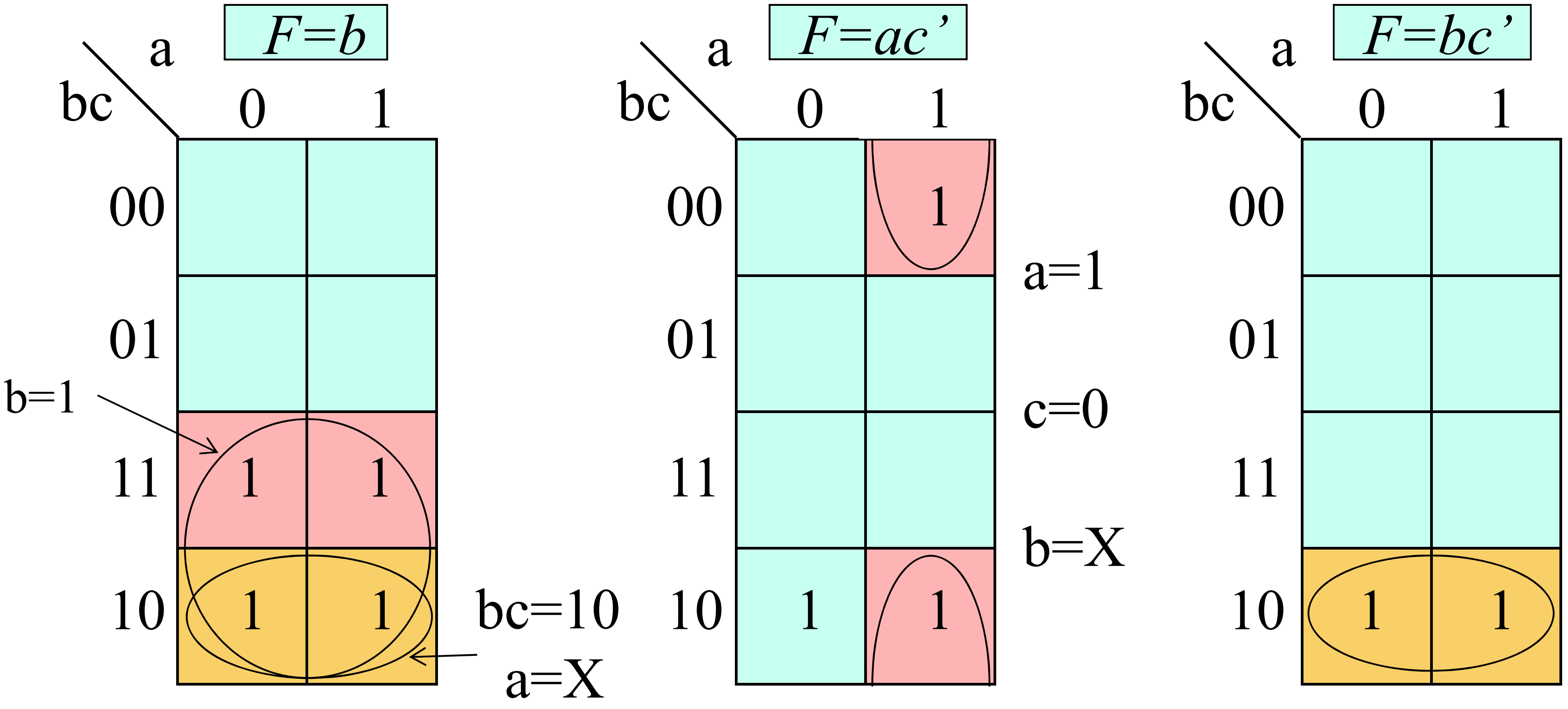


bc \ a	0	1
00	0 0	0 4
01	1 1	1 5
11	1 3	0 7
10	0 2	0 6

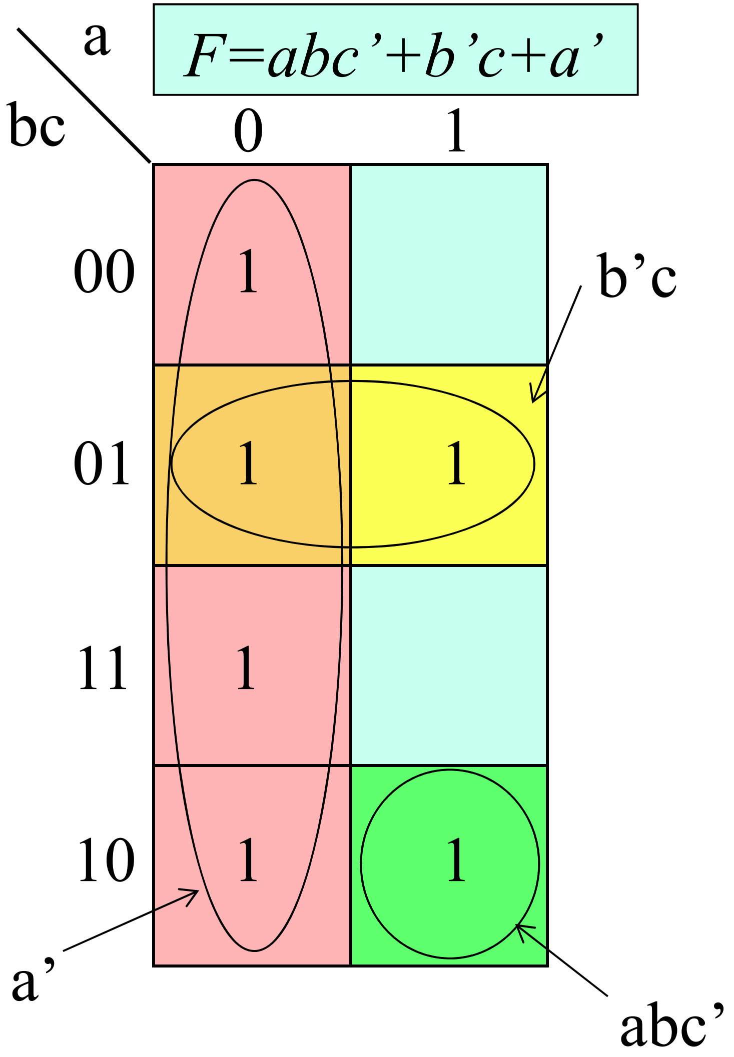
$$F = \Sigma m(1, 3, 5)$$
$$= \prod M(0, 2, 4, 6, 7)$$
$$= a'b'c + a'bc + ab'c$$
$$= a'c + b'c$$

2 and 3 Variables Karnaugh Maps (4/8)

Adjacency between each neighboring minterm



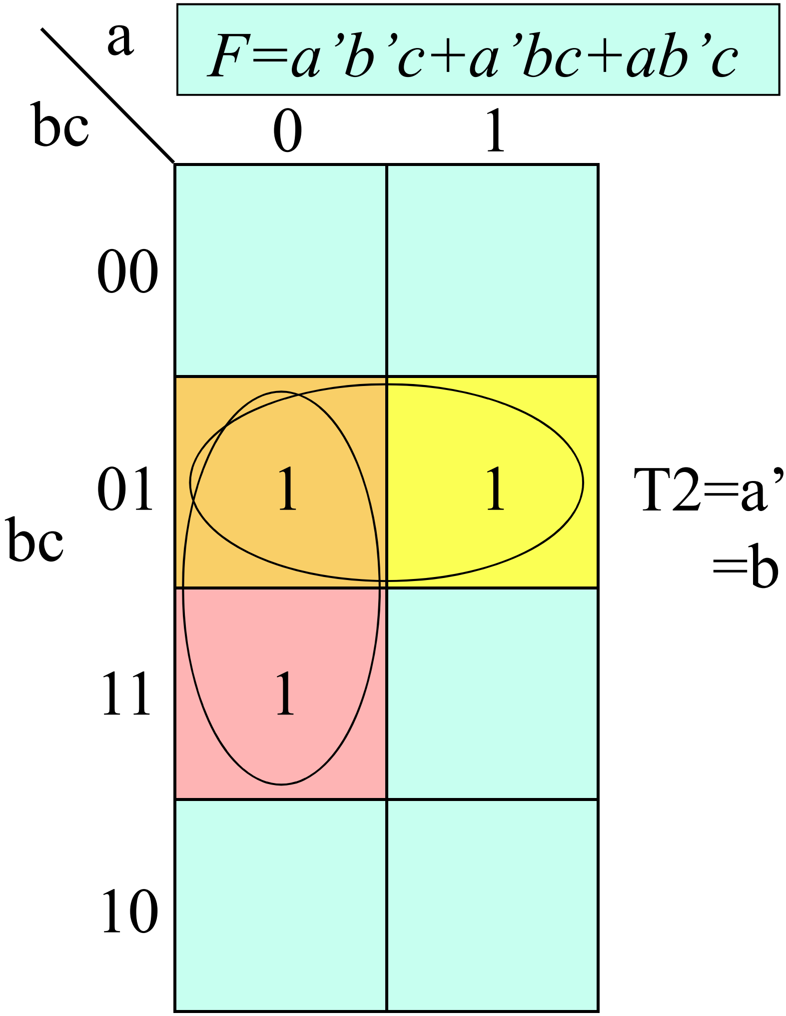
2 and 3 Variables Karnaugh Maps (5/8)



$F=abc'+b'c+a'$

bc \ a	0	1
00	1	
01	1	1
11	1	
10	1	1

2 and 3 Variables Karnaugh Maps (6/8)



F=a’b’c+a’bc+ab’c

bc \ a	0	1
00		
01	1	1
11	1	
10		

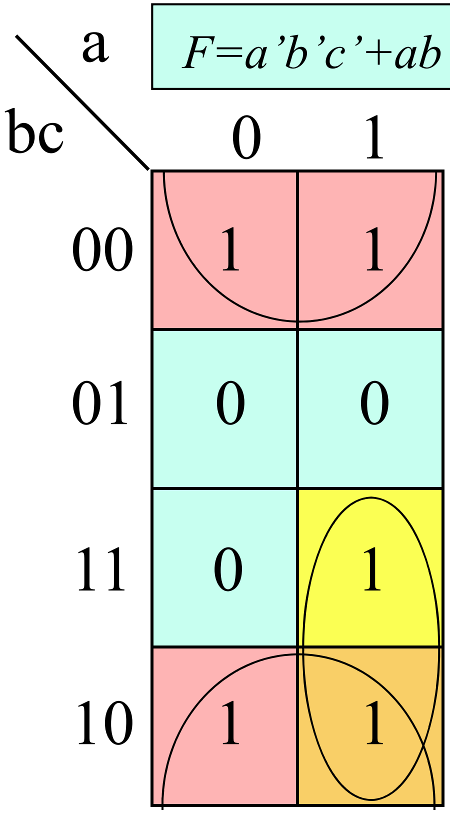
T1=a’b’c+a’bc
=a’c

T2=a’b’c+ab’c
=b’c

a’c “covers” minterms: (1, 3)=a’b’c + a’bc

2 and 3 Variables Karnaugh Maps (7/8)

F=a'b'c'+ab'c'+a'bc'+abc'+abc

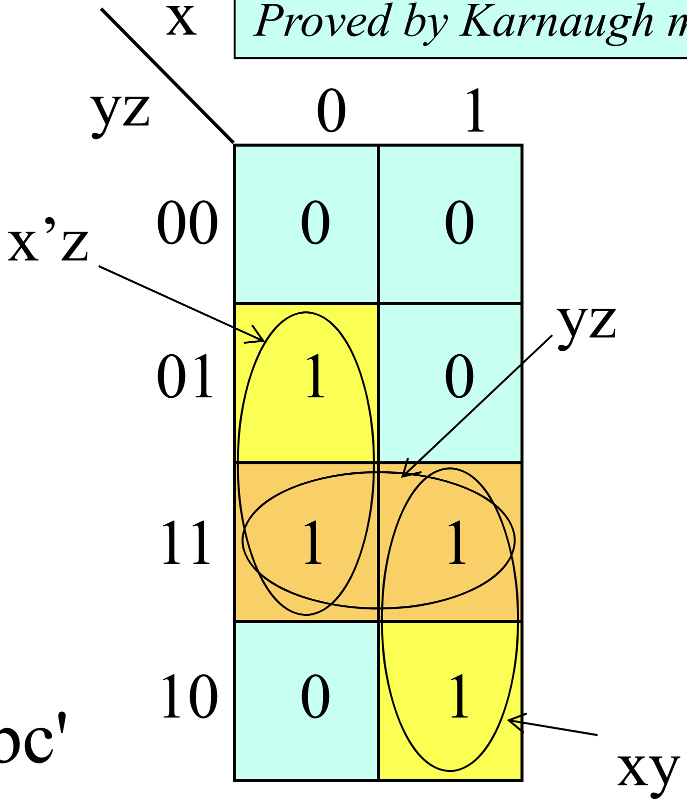


T1=a'b'c'+ab'c'
+a'bc'+abc'

=c'

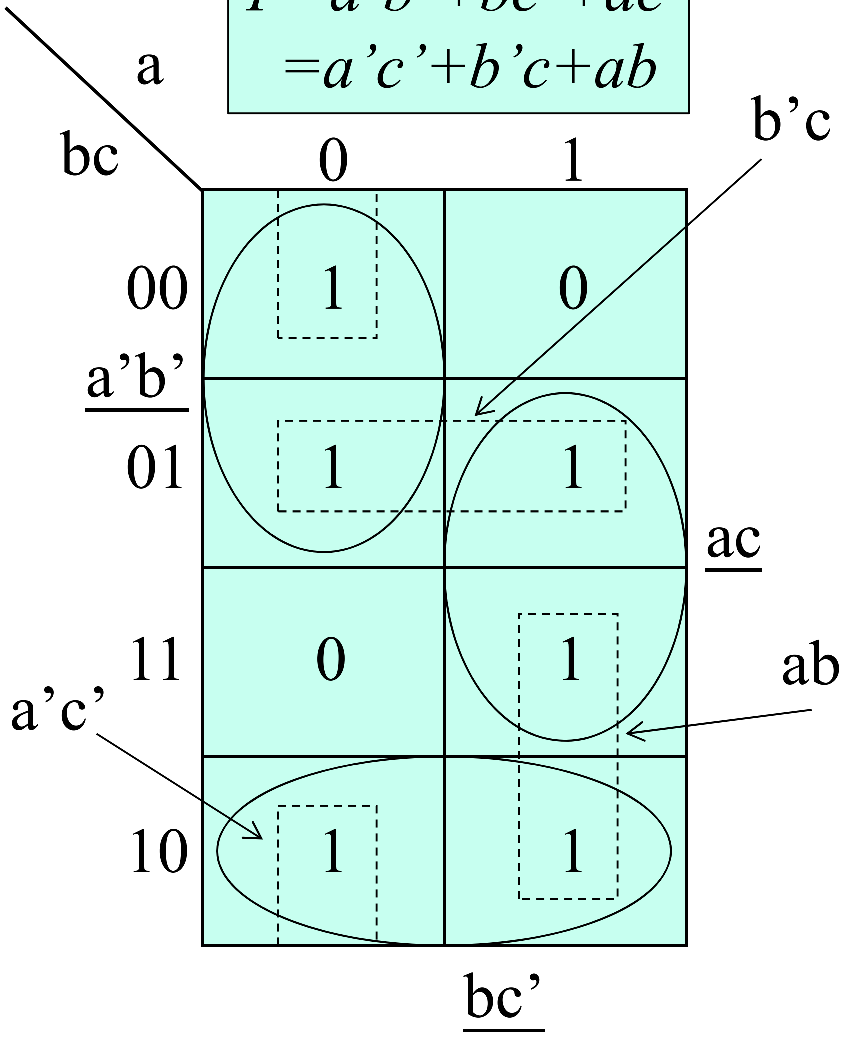
T2=abc+abc'

xy+x'z+yz=xy+x'z
Proved by Karnaugh map



2 and 3 Variables Karnaugh Maps (8/8)

$F = a'b' + bc' + ac$
$= a'c' + b'c + ab$



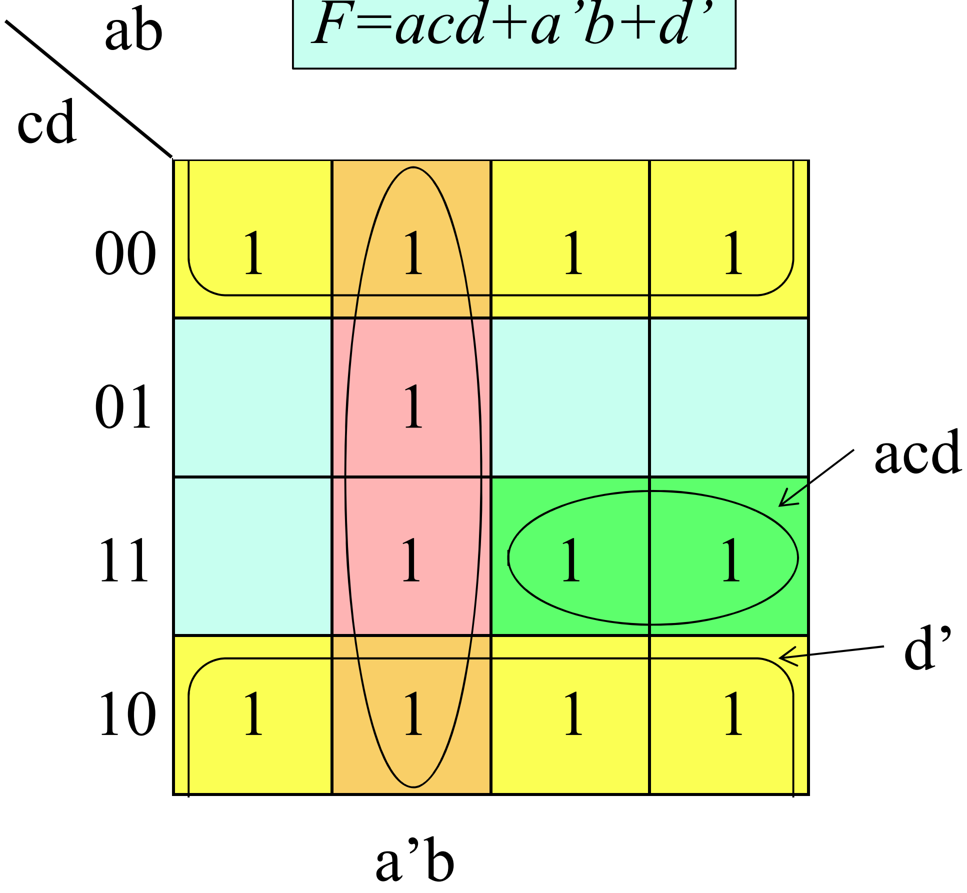
bc \ a	0	1
00	1	0
01	1	1
11	0	1
10	1	1

Two minimum forms

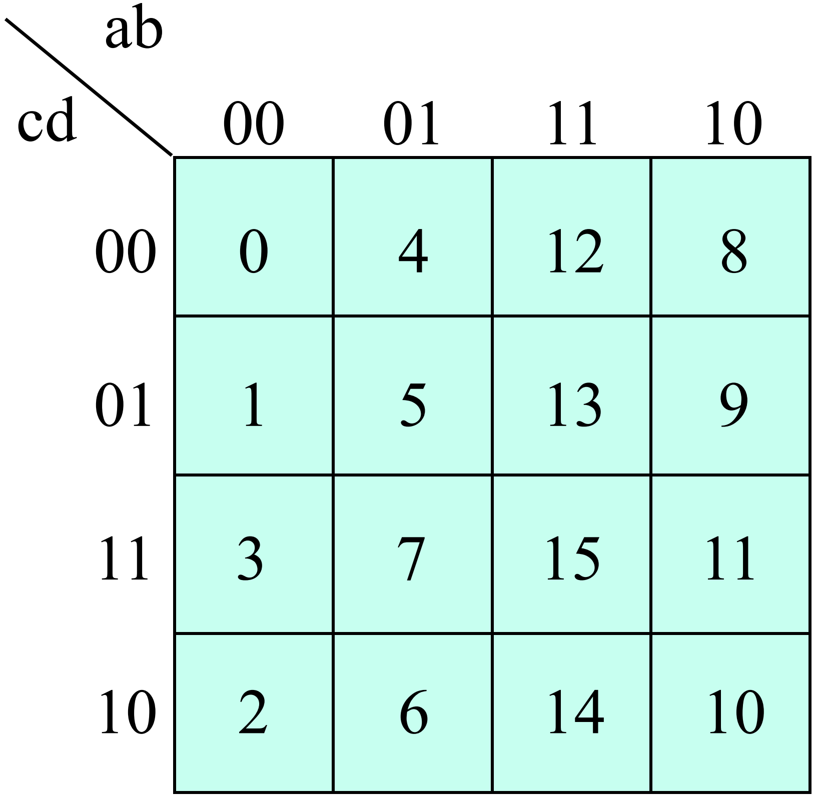
for the same function

4 Variables Karnaugh Maps (1/4)

F=acd+a'b+d'

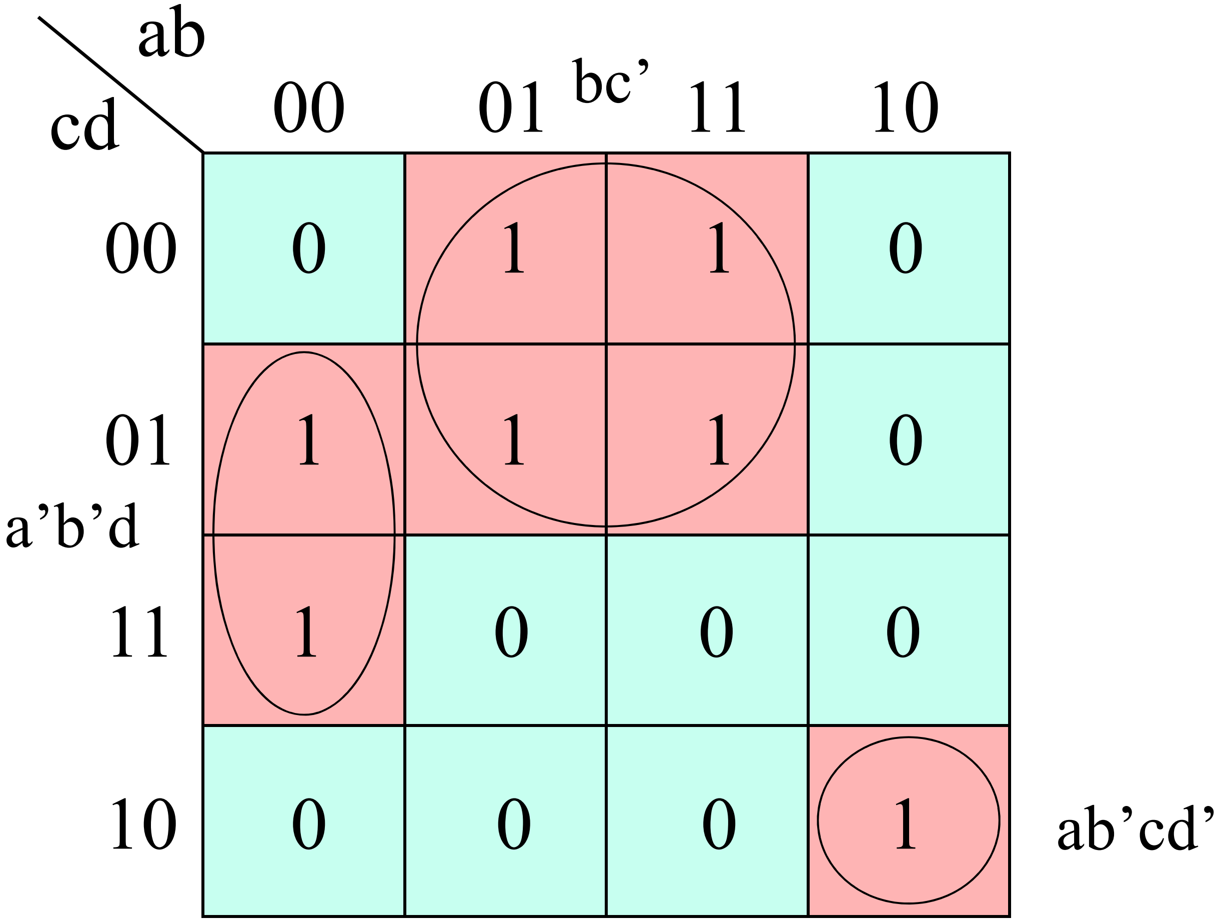


ab cd	00	01	11	10
00	1	1	1	1
01		1		
11		1	1	1
10	1	1	1	1



ab cd	00	01	11	10
00	0	4	12	8
01	1	5	13	9
11	3	7	15	11
10	2	6	14	10

4 Variables Karnaugh Maps (2/4)



cd \ ab	00	01	11	10
00	0	1	1	0
01	1	1	1	0
11	1	0	0	0
10	0	0	0	1

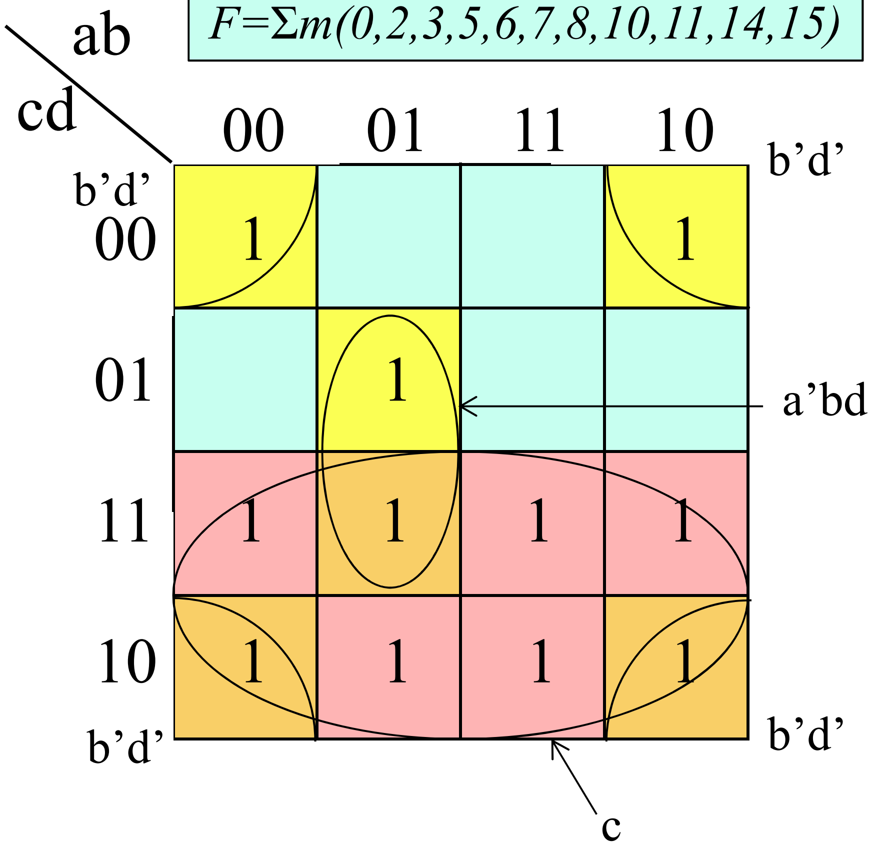
$F(a,b,c,d) = \Sigma(1,3,4,5,10,12,13)$

$F = bc' + a'b'd + ab'cd'$

4 Variables Karnaugh Maps (3/4)

$F = c + b'd' + a'bd$

$F = \Sigma m(0,2,3,5,6,7,8,10,11,14,15)$

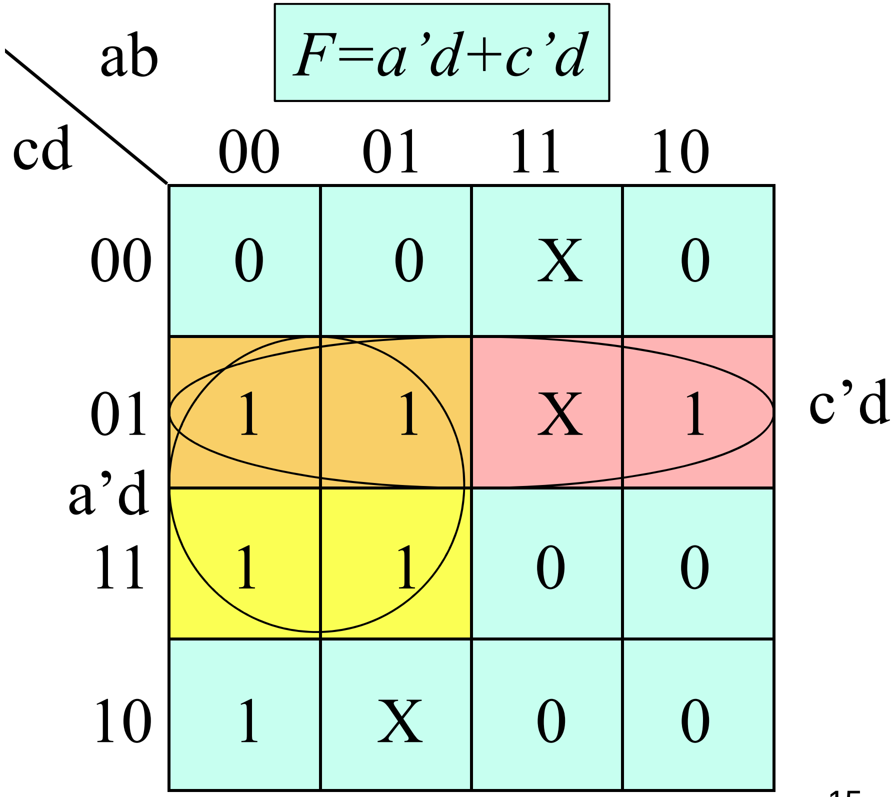


cd \ ab	00	01	11	10
00	1			1
01		1		
11	1	1	1	1
10	1	1	1	1

With Don't Care X

$$F = \Sigma m(1,3,5,7,9) + \Sigma d(6,12,13)$$

$F = a'd + c'd$



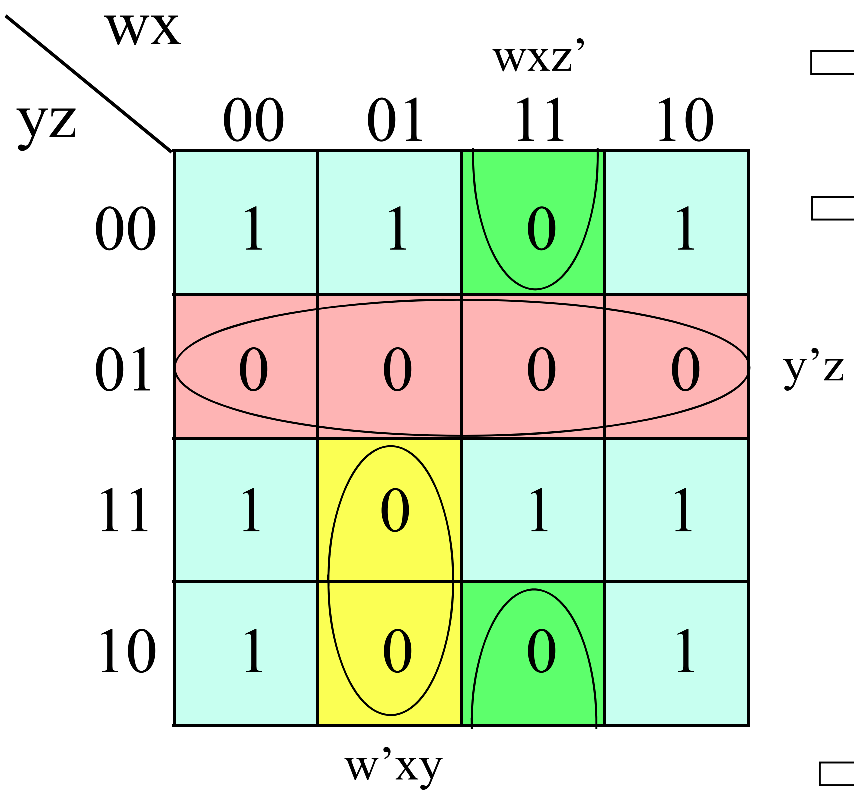
cd \ ab	00	01	11	10
00	0	0	X	0
01	1	1	X	1
11	1	1	0	0
10	1	X	0	0

4 Variables Karnaugh Maps (4/4)

$$F = \Sigma m\,(0,2,3,4,8,10,11,15)$$

$$\Rightarrow F' = \Sigma m\,(1,5,6,7,9,12,13,14)$$

$$\Rightarrow \underbrace{(F')'}_{F} = \prod M\,(1,5,6,7,9,12,13,14)$$



yz \ wx	00	01	11	10
00	1	1	0	1
01	0	0	0	0
11	1	0	1	1
10	1	0	0	1

$$F' = y'z + wxz' + w'xy$$

$$\Rightarrow F = (y+z')(w'+x'+z)(w+x'+y')$$

Minimum Forms with Essential Prime Implicants (1/4)

Implicant: Single element of the ON-SET or any group of elements that can be combined together in a K-Map.

Prime Implicant: Implicant that cannot be combined with another implicant to eliminate a term.

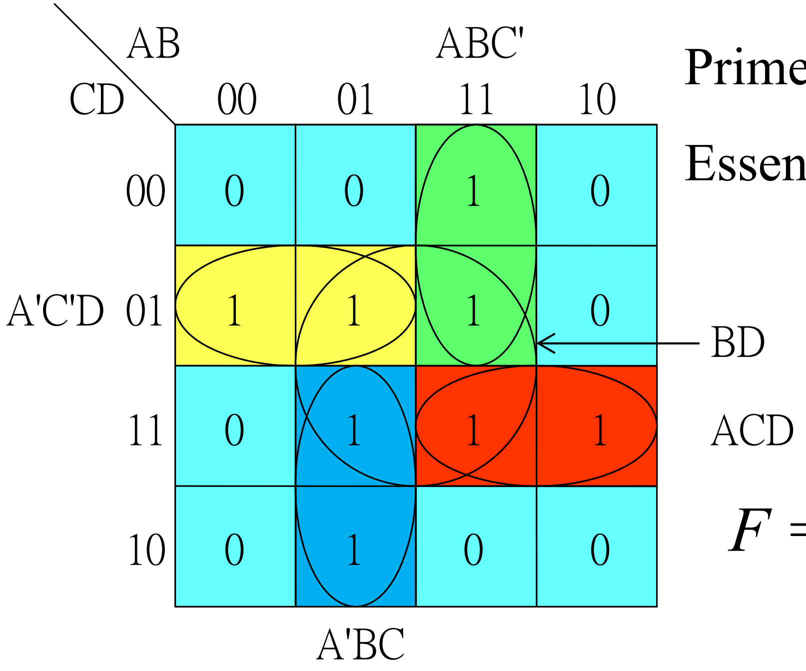
Essential Prime Implicant: If an element of the ON-SET is covered by a single prime implicant, the prime implicant is essential prime implicant.

Minimum Forms with Essential Prime Implicants (2/4)

1. Expand implicates into prime implicants.
2. Cover the ON-SET with as few prime implicants as possible.

Essential prime implicants participate in ALL possible covers.

Implicants:	m_1, m_5	m_{12},m_{13}	m_6,m_7	m_{11},m_{15}	
	A'C'D	ABC	A'BC	ACD	BD



CD \ AB	00	01	11	10
00	0	0	1	0
01	1	1	1	0
11	0	1	1	1
10	0	1	0	0

Prime Implicants: A'CD ABC ABC ACD BD

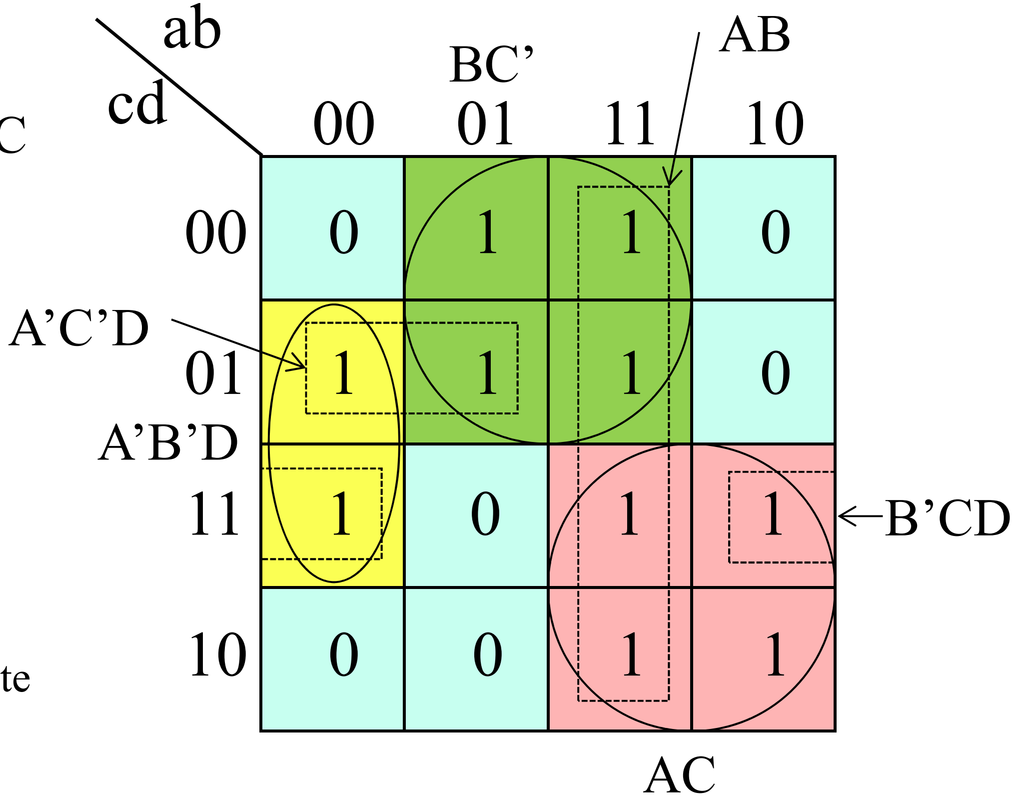
Essential Prime Implicants: A'CD ABC A'BC ACD

$$F = \underbrace{ABC' + ACD + A'BC + A'C'D}_{Minimim\ Cover} + BD$$

Minimum Forms with Essential Prime Implicants (3/4)

Prime Implicants: A'B'D, BC', AC, A'C'D, AB, B'CD

Essential Prime Implicants: BC', AC



cd \ ab	00	01	11	10
00	0	1	1	0
01	1	1	1	0
11	1	0	1	1
10	0	0	1	1

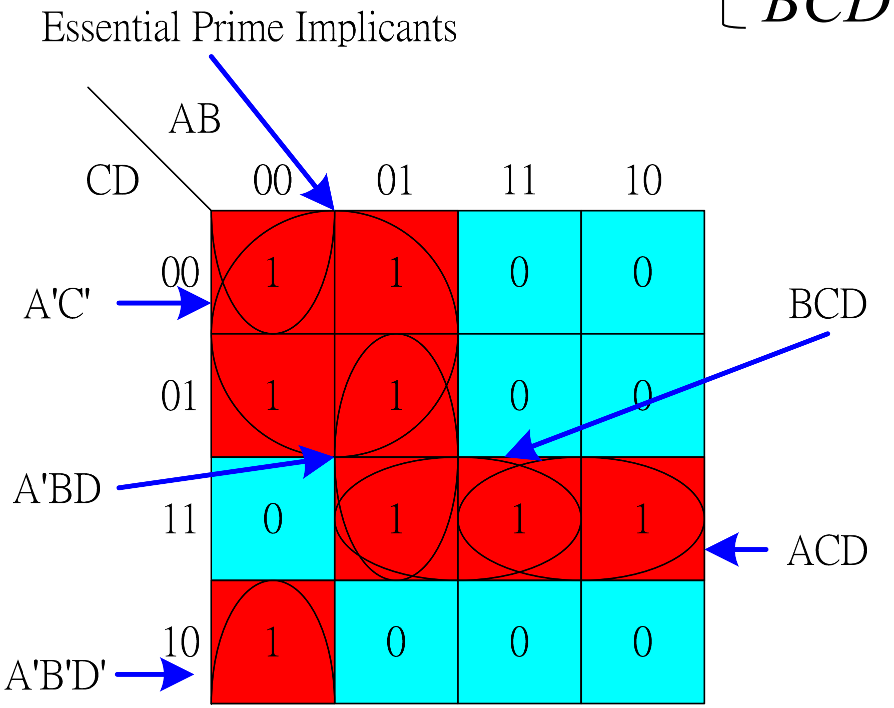
Essential Prime Implicants participate in all possible covers

$$F = \overbrace{BC'+AC} + A'C'D + B'CD$$

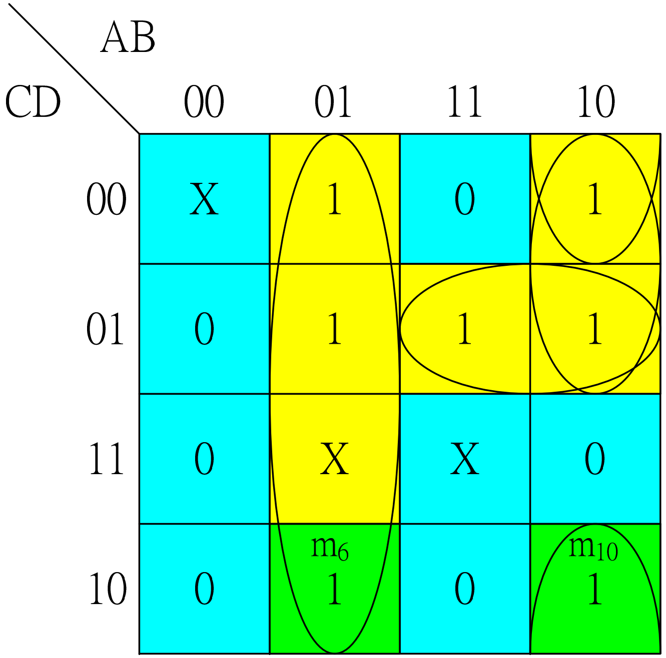
$F = BC'+AC+A'B'D$ ⇐ Minimum Cover

Minimum Forms with Essential Prime Implicants (4/4)

$$F = A'C' + A'B'D' + ACD + \left\{ \begin{matrix} A'BD \\ or \\ BCD \end{matrix} \right\}$$



CD \ AB	00	01	11	10
00	1	1	0	0
01	1	1	0	0
11	0	1	1	1
10	1	0	0	0



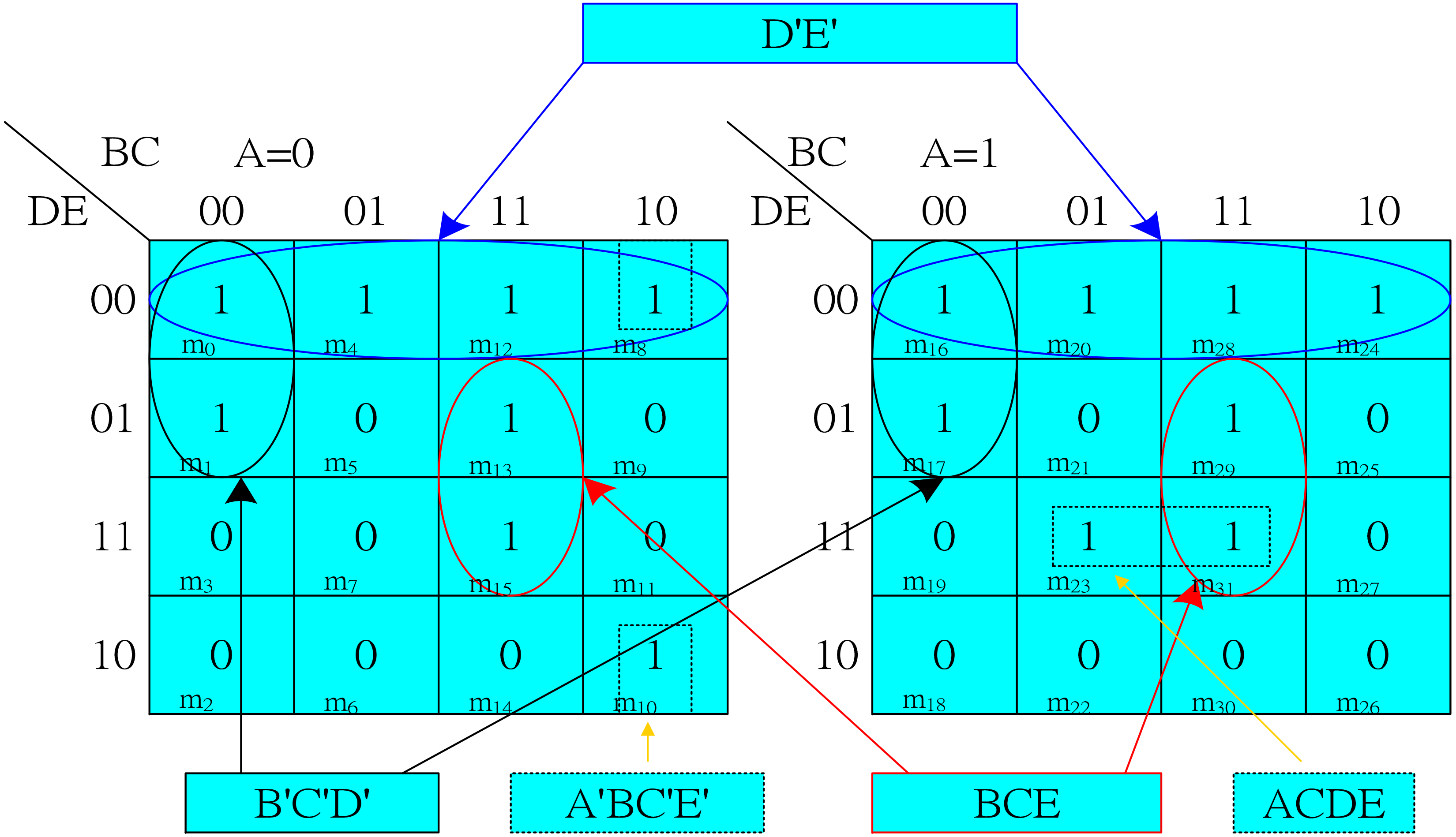
CD \ AB	00	01	11	10
00	X	1	0	1
01	0	1	1	1
11	0	X	X	0
10	0	m_6 1	0	m_{10} 1

Only m(6,10) are covered by only one prime implicant.

A minimum solution:

must contain all essential prime implicants

5 Variables Karnaugh Maps (1/2)



A=0

BC DE	00	01	11	10
00	1 m_0	1 m_4	1 m_{12}	1 m_8
01	1 m_1	0 m_5	1 m_{13}	0 m_9
11	0 m_3	0 m_7	1 m_{15}	0 m_{11}
10	0 m_2	0 m_6	0 m_{14}	1 m_{10}

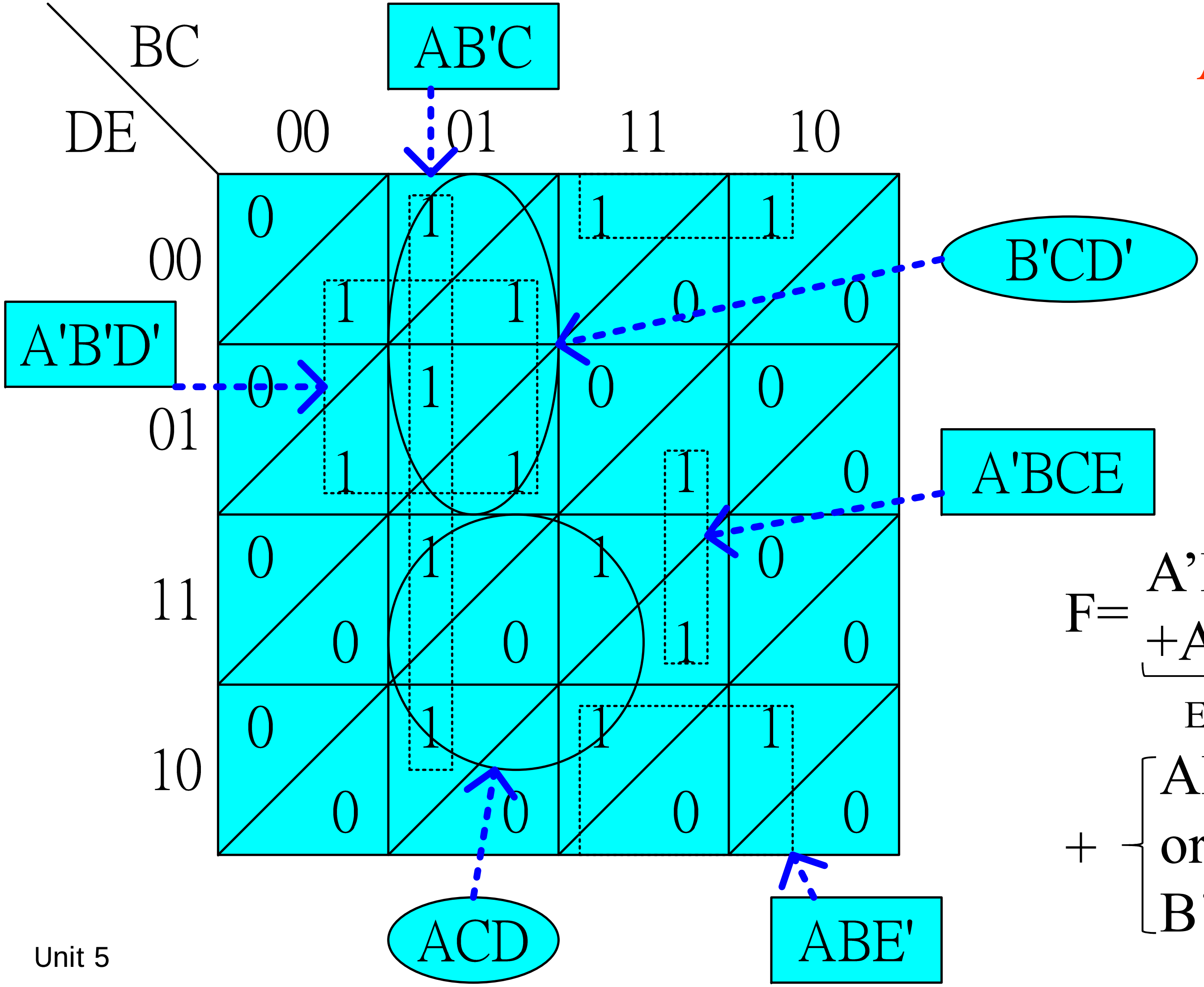
A=1

BC DE	00	01	11	10
00	1 m_{16}	1 m_{20}	1 m_{28}	1 m_{24}
01	1 m_{17}	0 m_{21}	1 m_{29}	0 m_{25}
11	0 m_{19}	1 m_{23}	1 m_{31}	0 m_{27}
10	0 m_{18}	0 m_{22}	0 m_{30}	0 m_{26}

$$F = D'E' + B'C'D' + BCE$$
$$+A'BC'E' + ACDE$$

5 Variables Karnaugh Maps (2/2)

$$F(A,B,C,D,E)=\Sigma\, m(0,1,4,5,13,15,20,21,22,23,24,26,28,30,31)$$



A=1/A=0

BC DE	00	01	11	10
00	0 / 1	1 / 1	1 / 0	1 / 0
01	0 / 1	1 / 1	0 / 1	0 / 0
11	0 / 0	1 / 0	1 / 1	0 / 0
10	0 / 0	1 / 0	1 / 0	1 / 0

$$F=\underbrace{A'B'D'+ABE'+ACD+A'BCE}_{\text{Essential Prime Implicants}}+\left\{\begin{matrix}AB'C\\ \text{or}\\ B'CD'\end{matrix}\right\}$$